

1 Normalization

Normalization is an algorithmic approach to **generating** a set of **relation schemas** from attributes and functional dependencies, that are not redundant (no unnecessary repetitions), and is able to represent all necessary information.

2 Functional Dependency

A **functional dependency** is a generalization of a key. It simply states that

in relation schema R, if A and B are **attribute sets**, $A \rightarrow B$ (A determines B) is a functional dependency, meaning that whenever 2 tuples in a relation over R are equal on attributes A, the tuples are equal on attributes B.

An example is ' $CPR \rightarrow \text{firstname}$ ' - if we know 'CPR', we know 'firstname', but not the other way around. Functional dependencies are decided by DB designers and are the **semantics** of the DB.

A **key constraint** is a functional dependency: ' $K \rightarrow R$ ', where the superkey determines all the attributes in a relation. Whenever the K attributes of the 2 tuples are equal, all the attributes are equal.

3 Boyce-Codd Normal Form (BCNF)

Boyce-Codd Normal Form (BCNF) is a DB decomposition requirement that **avoids most redundancy**, has **lossless join**, meaning that joining subrelations must yield the original relation. However, BCNF is **not always dependency preserving**, BCNF decompositions may violate functional dependencies.

We say that a relation schema R is in BCNF if all FDs $X \rightarrow Y$ associated with R are **trivial** ($Y \subseteq X$), e.g. $B \rightarrow B$, or X is a superkey for R, e.g. $B \rightarrow R$. In plain terms, this means that the only allowed FDs for a relation schema in BCNF are superkeys.

3.1 Attribute Set Closure

The **attribute set closure** X^+ , which is **all the attributes that X determine**, can be computed by repeatedly applying **Armstrongs axioms**

reflexivity $Y \subseteq X \implies X \rightarrow Y$ (find trivial FDs)

augmentation $X \rightarrow Y \implies AX \rightarrow AY$ (implicit \cup)

transitivity $X \rightarrow Y$ and $Y \rightarrow Z \implies X \rightarrow Z$

and derivations thereof. The attribute set closure is **sound**, meaning that it only generates correct FDs and it is **complete**, meaning that it generates *all* FDs.

3.2 Decomposition

A schema in BCNF can be created algorithmically by iteratively decomposing the original schema.

If there is a FD $X \rightarrow Y$ associated with a relation schema R that violates BCNF, the schema can be decomposed into subrelations XY and R-Y, e.g.

$$R = \text{name, CPR, pet-license-ID, pet-name}$$

FDs:

$CPR \rightarrow \text{name, pet-license-ID}$ and $\text{pet-license-ID} \rightarrow \text{pet-name}$. The latter violates BCNF, because $\text{pet-license-ID}^+ \neq R$

Decomposition:

$$\begin{aligned} P &= \{ \text{pet-license-ID, pet-name} \} \text{ (pet)} \\ O &= \{ \text{name, CPR, pet-license-ID} \} \text{ (owner)} \end{aligned}$$

3.3 Canonical Cover

A **canonical cover** for a set of FDs is an equivalent set of FDs **without redundancy**, meaning that there may not be extraneous attributes, and the left sides of FDs are unique. The canonical cover can be computed algorithmically with a **fixpoint algorithm**, that apply rules iteratively, until the result set does not change.

4 Third Normal Form (3NF)

All schemas in BCNF are in **Third Normal Form** (BCNF is stricter), but in contrast to BCNF it is always possible to find a 3NF decomposition that

is **dependency preserving** (FDs hold), has **lossless join** (joining yields original). However, 3NF decompositions may be **redundant**.

We say that a relation schema R is in 3NF if:

all FDs and their derivations (F^+) $X \rightarrow Y$ associated with R are either **trivial** or X is a superkey for R, or **each attribute in Y-X is contained in a candidate key for R**.

The **decomposition algorithm** for 3NF normalization works as follows:

```
for each FD X->Y in F+
    if no schema contains XY
        create new schema X,Y
if no schema contains candidate key for R
    create new schema with candidate key for R
```