

1 The absolute q -values are decreasing

1.0.1 The basic equations define the q -values

$$\begin{aligned} & \left(\frac{1}{1!}\right) \cdot q_4 + \left(\frac{1}{3!}\right) \cdot q_2 + \left(\frac{-1}{4!}\right) \cdot q_1 + \left(\frac{1}{5!}\right) \cdot q_0 = 0 \\ & \left(\frac{1}{1!}\right) \cdot q_6 + \left(\frac{1}{3!}\right) \cdot q_4 + \left(\frac{1}{5!}\right) \cdot q_2 + \left(\frac{-1}{6!}\right) \cdot q_1 + \left(\frac{1}{7!}\right) \cdot q_0 = 0 \\ & \left(\frac{1}{1!}\right) \cdot q_8 + \left(\frac{1}{3!}\right) \cdot q_6 + \left(\frac{1}{5!}\right) \cdot q_4 + \left(\frac{1}{7!}\right) \cdot q_2 + \left(\frac{-1}{8!}\right) \cdot q_1 + \left(\frac{1}{9!}\right) \cdot q_0 = 0 \\ & \left(\frac{1}{1!}\right) \cdot q_{10} + \left(\frac{1}{3!}\right) \cdot q_8 + \left(\frac{1}{5!}\right) \cdot q_6 + \left(\frac{1}{7!}\right) \cdot q_4 + \left(\frac{1}{9!}\right) \cdot q_2 + \left(\frac{-1}{10!}\right) \cdot q_1 + \left(\frac{1}{11!}\right) \cdot q_0 = 0 \end{aligned}$$

Note the q_1 -terms in the basic equations are negative.

In order to calculate the q -value of 8. You just multiply the equation with -1 and remove $-q_8$, and write q_8 on the other side of the equal-sign.

$$\left(\frac{1}{1!}\right) \cdot q_8 = -\left(\frac{1}{3!}\right) \cdot q_6 - \left(\frac{1}{5!}\right) \cdot q_4 - \left(\frac{1}{7!}\right) \cdot q_2 - \left(\frac{1}{8!}\right) \cdot q_1 - \left(\frac{-1}{9!}\right) \cdot q_0$$

and the same way, when you want to calculate q_{10}

$$\left(\frac{1}{1!}\right) \cdot q_{10} = -\left(\frac{1}{3!}\right) \cdot q_8 - \left(\frac{1}{5!}\right) \cdot q_6 - \left(\frac{1}{7!}\right) \cdot q_4 - \left(\frac{1}{9!}\right) \cdot q_2 - \left(\frac{-1}{10!}\right) \cdot q_1 - \left(\frac{1}{11!}\right) \cdot q_0$$

$q_0 = 1$, $q_1 = \frac{1}{2}$ and $q_2 = \frac{1}{12}$.

I used **the equations of the increasing numerator** to demonstrate, that q -values of uneven numbers greater than 1 are zero, that q -values of numbers, which are divisible with 4 are negative and q -values of the even (but not divisible with 4) numbers are positive.

Last, but important for this work: I demonstrated another (recursiv) algorithm to calculate the q -values from all earlier calculated q -values than **the basic equations**.

$$\begin{aligned} -5 \cdot q_4 &= q_2 \cdot q_2 \\ -7 \cdot q_6 &= q_2 \cdot q_4 + q_4 \cdot q_2 \\ -9 \cdot q_8 &= q_2 \cdot q_6 + q_4 \cdot q_4 + q_6 \cdot q_2 \\ -11 \cdot q_{10} &= q_2 \cdot q_8 + q_4 \cdot q_6 + q_6 \cdot q_4 + q_8 \cdot q_2 \\ -(2n+1) \cdot q_{(2n)} &= q_2 \cdot q_{(2n-2)} + q_2 \cdot q_{(2n-4)} + \dots + q_{(2n-4)} \cdot q_4 + q_{(2n-2)} \cdot q_2 \end{aligned}$$

We will use these equations to prove, that the absolute value of the q -values are decreasing.

First I will show an example

$$|q_8| > |q_{10}|$$

$$-9 \cdot q_8 = q_2 \cdot q_6 + q_4 \cdot q_4 + q_6 \cdot q_2$$

Note an interesting thing about these equations: if you on the left side of the equal-sign have the q -value of a number divisible with 4, which we have learnt above means, the q -value is negative, then on the right side all the products necessary to calculate it are positive.

$-9 \cdot q_8$ must be equal to $9 \cdot |q_8|$ // We can write:

$$9 \cdot |q_8| = q_2 \cdot q_6 + q_4 \cdot q_4 + q_6 \cdot q_2$$

$$-11 \cdot q_{10} = q_2 \cdot q_8 + q_4 \cdot q_6 + q_6 \cdot q_4 + q_2 \cdot q_8$$

q_{10} must be positive since 10 is not divisible with 4, on the right side of the equals sign all the products must be negative since in every one product there is one and only one q -value of a number divisible with 4.

If we place these negative q -values on the right between vertical bars

We can write.

$$11 \cdot q_{10} = q_2 \cdot |q_8| + |q_4| \cdot q_6 + q_6 \cdot |q_4| + |q_8| \cdot q_2$$

If we want to decide, whether $|q_8| > |q_{10}|$ or $|q_8| < |q_{10}|$, we have two problem:

The first: there are 9 $|q_8|$ but 11 q_{10}

The second there are 3 products in the equation of $|q_8|$ but 4 products in the equation of q_{10} .

Both can be remedied by adding $2 \cdot |q_8|$ on both side of the equals sign.

$$11 \cdot |q_8| = 2 \cdot |q_8| + q_2 \cdot q_6 + |q_4| \cdot |q_4| + q_6 \cdot q_2$$

$$11 \cdot q_{10} = q_2 \cdot |q_8| + |q_4| \cdot q_6 + q_6 \cdot |q_4| + |q_8| \cdot q_2$$

We now make this assumption: $q_2 > |q_4| > q_6 > |q_8|$.

We have calculated that in maple or in hand.

All the products in the equation of q_{10} have a factor smaller than the corresponding factor in the equation of $|q_8|$ the other factor being the same.

Take the product $|q_4| \cdot |q_4|$ from the equation of $|q_8|$ and $|q_4| \cdot q_6$ from the equation of q_{10} . q_6 is smaller than $|q_4|$.

What about the fabricated first term in the equation of $2 \cdot |q_8|$ compared with $q_2 \cdot |q_8|$ from the equation of q_{10} ?

Well $q_2 = \frac{1}{12}$ is 24 times smaller than 2

We can conclude:

$$|q_8| > q_{10}$$

Generalized these equations can be written so:

$$(2 \cdot t + 1) \cdot |q_{(2 \cdot t)}| = \sum_{s=1}^{t-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}|$$

and

$$(2 \cdot t + 3) \cdot |q_{(2 \cdot t + 2)}| = \sum_{s=1}^t |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t + 2 - 2 \cdot s)}|$$

In the example above we placed the q -values of the numbers divisible with 4 between vertical bars because they are negative. From now on we do that with all q -values, since it makes no difference.

There is a product more in the second sum.

As we did in the example above, we add $2 \cdot |q_{(2 \cdot t)}|$ and get:

$$(2 \cdot t + 3) \cdot |q_{(2 \cdot t)}| = 2 \cdot |q_{(2 \cdot t)}| + \sum_{s=1}^{t-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}|$$

$$(2 \cdot t + 3) \cdot |q_{(2 \cdot t)}| = 2 \cdot |q_{(2 \cdot t)}| + \sum_{s=1}^{t-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}|$$

$$(2 \cdot t + 3) \cdot |q_{(2 \cdot t + 2)}| = \sum_{s=1}^t |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t + 2 - 2 \cdot s)}|$$

We take the last product (the first and last product are equal) out of this sum and place it in front of the sum and get:

$$(2 \cdot t + 3) \cdot |q_{(2 \cdot t + 2)}| = |q_{(2)}| \cdot |q_{(2 \cdot t)}| + \sum_{s=1}^{t-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t + 2 - 2 \cdot s)}|$$

All the products in the equation of $|q_{(2 \cdot t+2)}|$ have a factor smaller than the corresponding factor in the equation of $|q_{(2 \cdot t)}|$ the other factor being the same.

We have shown, $|q_8| > |q_{10}|$.

Then we can carry on and let t be 5 so $|q_{10}| > |q_{12}|$ and so on.//

We can conclude:

$$|q_{(2 \cdot t)}| > |q_{(2 \cdot t+2)}|$$

The q -values are decreasing.

Q.E.D.