1 Genocchis numbers

In this work I want to schow that

$$g(2n) = (2n)! \cdot q(2n) \cdot 2 \cdot (2^{2 \cdot n} - 1)$$

are whole numbers.
$g(2n)$ is a funktion of the even numbers

The numbers is called Genocchi's numbers:
$g(2) = 1,\ g(4) = -1,\ g(6) = 3,\ g(8) = -17,\ g(10) = 155$. The sequence grows rapidly.
It is easy to show that this row of numbers diverge:

$$|\left(\frac{g(2n+2)}{g(2n)}\right)| = |\left(\frac{(2n+2)! \cdot q(2n+2) \cdot 2 \cdot (2^{2 \cdot n+2} - 1)}{(2n)! \cdot q(2n) \cdot 2 \cdot (2^{2 \cdot n} - 1)}\right)|$$

There is no upper limit.
Demonstration:

$$|\frac{g(2n+2)}{g(2n)}| > \left(\frac{n^2}{4}\right)$$

and
If you want the fraction bigger than K (a big number:), you must choose your $2n$ bigger than $4 \cdot K^{0.5}$
In the following, we will make the fraction more and more simple. We will do this in a way, that secure it always gets smaller. Then we can end up saying, this fraction is smaller than the original, and has nevertheless no upper limit.

$$|\left(\frac{(2n+2)! \cdot q(2n+2) \cdot 2 \cdot (2^{2 \cdot n+2} - 1)}{(2n)! \cdot q(2n) \cdot 2 \cdot (2^{2 \cdot n} - 1)}\right)| =$$

$$\left(\frac{(2n+1) \cdot (2n+2)}{|\frac{q(2n)}{q(n+2)}|}\right) \cdot \left(\frac{2 \cdot (2^{2 \cdot n+2} - 1)}{2 \cdot (2^{2 \cdot n} - 1)}\right) >$$

$$\left(\frac{(2n+1) \cdot (2n+2)}{60}\right) \cdot \left(\frac{(2^{2 \cdot n+2} - 1)}{(2^{2 \cdot n} - 1)}\right)$$

Why 60?
In an earlier work I have demonstrated that $|\frac{q(2n)}{q(2n+2)}|$ is a decreasing function of the even numbers with a limit near $(2 \cdot \pi)^2 = 39.478....$
$|\frac{q(2n)}{q(2n+2)}|$: 60, 42, 40, 39.6, 39.508, 39.486,...
$q(2n)$ is defined in the beginning of the next chapter.
I then choose the first and so the greatest value $|\frac{q(2)}{q(4)}| = 60$ to secure the result smaller than the original value of $|\frac{g(2n+2)}{g(2n)}|$.

$$\left(\frac{(2n+1) \cdot (2n+2)}{60}\right) \cdot \left(\frac{(2^{2 \cdot n+2} - 1)}{(2^{2 \cdot n} - 1)}\right) >$$

$$\left(\frac{(2n)^2}{60}\right)\cdot\left(\frac{(2^{2\cdot n+2}-1)}{(2^{2\cdot n}-1)}\right) > \left(\frac{(2n)^2}{60}\right)\cdot\left(\frac{(2^{2\cdot n+2})}{(2^{2\cdot n})}\right)$$

Is $\frac{(2^{2\cdot n+2})}{(2^{2\cdot n})}$ really smaller than $\frac{(2^{2\cdot n+2}-1)}{(2^{2\cdot n}-1)}$?
Yes, look!

$$\frac{(2^{2\cdot n+2}-1)}{(2^{2\cdot n}-1)} - \frac{(2^{2\cdot n+2})}{(2^{2\cdot n})} = \frac{(2^{2\cdot n+2}-1)\cdot(2^{2\cdot n}) - (2^{2\cdot n+2})\cdot(2^{2\cdot n}-1)}{(2^{2\cdot n}-1)\cdot(2^{2\cdot n})}$$

$$= \frac{(2^{4\cdot n+2}-2^{2\cdot n}) - (2^{4\cdot n+2}-2^{2\cdot n+2})}{(2^{2\cdot n}-1)\cdot(2^{2\cdot n})} = \frac{2^{2\cdot n}\cdot(2^2-1)}{(2^{2\cdot n}-1)\cdot(2^{2\cdot n})} = \frac{(2^2-1)}{(2^{2\cdot n}-1)}$$

The difference is positive.
We can safely replace $\frac{(2^{2\cdot n+2}-1)}{(2^{2\cdot n}-1)}$ with $\frac{(2^{2\cdot n+2})}{(2^{2\cdot n})} = 2^2$ and then we have:

$$\left(\frac{(2n)^2}{60}\right)\cdot\left(\frac{(2^{2\cdot n+2})}{(2^{2\cdot n})}\right) = \left(\frac{(2n)^2\cdot 2^2}{60}\right) = \left(\frac{(2n)^2}{15}\right) > \left(\frac{4\cdot n^2}{16}\right) = \left(\frac{n^2}{4}\right)$$

At the end:

$$|\frac{g(2n+2)}{g(2n)}| > 0.25\cdot n^2$$

When is $|\frac{g(2n+2)}{g(2n)}|$ larger than K, K being a big number (10, 100, 1000)?

$$0.25\cdot n^2 > K$$

implies

$$n > 2\cdot K^{0.5}$$

and

$$2n > 4\cdot K^{0.5}$$

and K=10 implies

$$2n > 4\cdot 10^{0.5} > 4\cdot 3 = 12$$

So we get:

$$|\frac{g(14)}{g(12)}| > 10$$

And K=100 implies

$$2n > 4\cdot 100^{0.5} = 4\cdot 10 = 40$$

We get:

$$|\frac{g(42)}{g(40)}| > 100$$

and K=1000 implies

$$2n > 4\cdot 1000^{0.5} > 4\cdot 10\cdot 3 = 120$$

We get:

$$|\frac{g(122)}{g(120)}| > 1000$$

$$|g(2n+2)| > 0.25 \cdot n^2 \cdot |g(2n)|$$
$$|g(10)| > 0.25 \cdot 4^2 \cdot |g(8)| = 4 \cdot |g(8)|$$
$$|g(22)| > 0.25 \cdot 10^2 \cdot |g(20)| = 25 \cdot |g(20)|$$
$$|g(202)| > 0.25 \cdot 100^2 \cdot |g(200)| = 2500 \cdot |g(200)|$$

Q.E.D.

1.1 Prerequisites

1.1.1 Punkt 1: My basic equations

$$\left(\frac{1}{1!}\right) \cdot q4 \cdot (1) + \left(\frac{1}{3!}\right) \cdot q2 \cdot (1) + \left(\frac{-1}{4!}\right) \cdot q1 \cdot (1) + \left(\frac{1}{5!}\right) \cdot q0 \cdot (1) = 0$$
$$\left(\frac{1}{1!}\right) \cdot q6 \cdot (1) + \left(\frac{1}{3!}\right) \cdot q4 \cdot (1) + \left(\frac{1}{5!}\right) \cdot q2 \cdot (1) + \left(\frac{-1}{6!}\right) \cdot q1 \cdot (1) + \left(\frac{1}{7!}\right) \cdot q0 \cdot (1) = 0$$
$$\left(\frac{1}{1!}\right) \cdot q8 \cdot (1) + \left(\frac{1}{3!}\right) \cdot q6 \cdot (1) + \left(\frac{1}{5!}\right) \cdot q4 \cdot (1) + \left(\frac{1}{7!}\right) \cdot q2 \cdot (1) + \left(\frac{-1}{8!}\right) \cdot q1 \cdot (1) + \left(\frac{1}{9!}\right) \cdot q0 \cdot (1) = 0$$

In an earlier work I defined a function of the natural numbers and zero with the help of a recursiv algoritme, which must emerge from the examples above.
$q8$ is the sum of the rest with opposite sign.
$q0 = 1,\ q1 = (\frac{1}{2})$ og $q2 = (\frac{1}{12})$
q-values of uneven numbers greater than 1 are zero and therefore omitted.
The q-values of the even numbers are converging to zero very fast.
In an earlier work I have demonstrated, that $|\frac{q(2n)}{q(2n+2)}|$ converge pretty fast to a limit, seemingly $(2 \cdot pi)^2$, but until now I have not been able to confirm this.
$(2 \cdot pi)^2 = 39.478... : 60, 42, 40, 39.6, 39.5, ...$
I used these 'basic equations' to prove these formulas, where I transform a sum of consecutively natural numbers raised to the power of n to a sum of powers with decreasing (decreasing with n in qn) eksponents starting with z^{n+1}

$$\sum_{z=1}^{6} z^8 = \left(\frac{8!}{1!}\right) \cdot q8 \cdot (z^1) + \left(\frac{8!}{3!}\right) \cdot q6 \cdot (z^3) + \left(\frac{8!}{5!}\right) \cdot q4 \cdot (z^5) + \left(\frac{8!}{7!}\right) \cdot q2 \cdot (z^7) + \left(\frac{8!}{8!}\right) \cdot q1 \cdot (z^8) + \left(\frac{8!}{9!}\right) \cdot q0 \cdot (z^9)$$

For $z = 6$ this formula calculates the sum $1^8 + 2^8 + 3^8 + 4^8 + 5^8 + 6^8$

1.1.2 Punkt 2: My equations of the increasing numerator

$$\left(\frac{1+5}{1!}\right) q4\cdot(1)+\left(\frac{3}{3!}\right) q2\cdot(1)+\left(\frac{-2}{4!}\right) q1\cdot(1)+\left(\frac{1}{5!}\right) q0\cdot(1)=0$$
$$\left(\frac{1+7}{1!}\right) q6\cdot(1)+\left(\frac{5}{3!}\right) q4\cdot(1)+\left(\frac{3}{5!}\right) q2\cdot(1)+\left(\frac{-2}{6!}\right) q1\cdot(1)+\left(\frac{1}{7!}\right) q0\cdot(1)=0$$
$$\left(\frac{1+9}{1!}\right) q8\cdot(1)+\left(\frac{7}{3!}\right) q6\cdot(1)+\left(\frac{5}{5!}\right) q4\cdot(1)+\left(\frac{3}{7!}\right) q2\cdot(1)+\left(\frac{-2}{8!}\right) q1\cdot(1)+\left(\frac{1}{9!}\right) q0\cdot(1)=0$$

The increasing numerators are seen read from right to left. Note the last fraction gets an even numerator: $(\frac{1+5}{1!})\cdot q4\cdot(1)$, $(\frac{1+7}{1!})\cdot q6\cdot(1)$ og $(\frac{1+9}{1!})\cdot q8\cdot(1)$
In an earlier work I have demonstrated the existence of these abovementioned equations.
I used the equations of the increasing numerator to demonstrate, that q-values of uneven numbers greater than 1 are zero, that q-values of numbers, which are divisible with 4 are negative and q-values of the even (but not divisible with 4) numbers are positive.
Last, but importent for this work: I demonstrated another (recursiv) algorithm to calculate the q-values from all earlier calculated q-values than **the basic equations**.

$$-5\cdot q4 = q2\cdot q2$$
$$-7\cdot q6 = q2\cdot q4 + q4\cdot q2$$
$$-9\cdot q8 = q2\cdot q6 + q4\cdot q4 + q6\cdot q2$$
$$-11\cdot q10 = q2\cdot q8 + q4\cdot q6 + q6\cdot q4 + q8\cdot q2$$
$$-(2n+1)\cdot q(2n) = q2\cdot q(2n-2) + q4\cdot q(2n-4) + ... + q(2n-4)\cdot q4 + q(2n+2)\cdot q2$$

When $2n$ in the calculation of $-(2n+1)\cdot q(2n)$ is divisible with 4 as in the term $-(2\cdot 4+1)\cdot q(8)$, we will meet this alonestanding **middleterm** $qn\cdot qn$ $(q4\cdot q4)$. Otherwise all terms $(q2q6)$ will have a counterpart in the other end of the equation the factors only swapped $(q6q2)$

1.1.3 The mirrorequation nr. 1

We have hier below 3 mirrorequation nr. 1. I have calculated the left side in all 3 equations to be zero.

$$\left(\frac{1}{1!}\right)\cdot q6\cdot(2^6)+\left(\frac{1}{3!}\right)\cdot q4\cdot(2^4)+\left(\frac{1}{5!}\right)\cdot q2\cdot(2^2)+\left(\frac{-1}{6!}\right)\cdot q1\cdot(2^1)+\left(\frac{1}{7!}\right)\cdot q0\cdot(2^0)=0$$
$$\left(\frac{1}{1!}\right)\cdot q4\cdot(2^4)+\left(\frac{1}{3!}\right)\cdot q2\cdot(2^2)+\left(\frac{-1}{4!}\right)\cdot q1\cdot(2^1)+\left(\frac{1}{5!}\right)\cdot q0\cdot(2^0)=0$$
$$\left(\frac{1}{1!}\right)\cdot q2\cdot(2^2)+\left(\frac{-1}{2!}\right)\cdot q1\cdot(2^1)+\left(\frac{1}{3!}\right)\cdot q0\cdot(2^0)=0$$

I have named them so, because the eksponents in the term $q(2n)\cdot 2^{2\cdot n}$ equals $2n$, increase with $2n$, whereas the eksponents decrease with $2n$ in the formula of powers in my former work.

$$\sum_{z=1}^{6} z^8=\left(\frac{8!}{1!}\right)\cdot q8\cdot(z^1)+\left(\frac{8!}{3!}\right)\cdot q6\cdot(z^3)+\left(\frac{8!}{5!}\right)\cdot q4\cdot(z^5)+\left(\frac{8!}{7!}\right)\cdot q2\cdot(z^7)+\left(\frac{8!}{8!}\right)\cdot q1\cdot(z^8)+\left(\frac{8!}{9!}\right)\cdot q0\cdot(z^9)$$

I multiply the top equation with $2^2\cdot q2$, the middle with $2^4\cdot q4$ og the last with $2^6\cdot q6$.

I intend now to show, that I with this new triangle will be able to construct the next mirrorequation nr. 1.

$$\left(\frac{1}{1!}\right)2^6q6\cdot 2^2q2+\left(\frac{1}{3!}\right)2^4q4\cdot 2^2q2+\left(\frac{1}{5!}\right)2^2q2\cdot 2^2q2+\left(\frac{-1}{6!}\right)2^1q1\cdot 2^2q2+\left(\frac{1}{7!}\right)2^0q0\cdot 2^2q2=0$$
$$\left(\frac{1}{1!}\right)2^4q4\cdot 2^4q4+\left(\frac{1}{3!}\right)2^2q2\cdot 2^4q4+\left(\frac{-1}{4!}\right)2^1q1\cdot 2^4q4+\left(\frac{1}{5!}\right)2^0q0\cdot 2^4q4=0$$
$$\left(\frac{1}{1!}\right)2^6q2\cdot 2^6q6+\left(\frac{-1}{2!}\right)2^1q1\cdot 2^6q6+\left(\frac{1}{3!}\right)2^0q0\cdot 2^6q6=0$$

If we now remember that $q0=1$ og $q1=\frac{1}{2}$, we can add the 2 last terms in the equations and get:

$$\left(\frac{1}{1!}\right)2^6q6\cdot 2^2q2+\left(\frac{1}{3!}\right)2^4q4\cdot 2^2q2+\left(\frac{1}{5!}\right)2^2q2\cdot 2^2q2+\left(\frac{-6}{7!}\right)\cdot 2^2q2=0$$
$$\left(\frac{1}{1!}\right)2^4q4\cdot 2^4q4+\left(\frac{1}{3!}\right)2^2q2\cdot 2^4q4+\left(\frac{-4}{5!}\right)\cdot 2^4q4=0$$
$$\left(\frac{1}{1!}\right)2^6q2\cdot 2^6q6+\left(\frac{-2}{3!}\right)\cdot 2^6q6=0$$

We multiply the powers in every term and get

$$\left(\frac{1}{1!}\right)2^8\cdot q6q2+\left(\frac{1}{3!}\right)2^6\cdot q4q2+\left(\frac{1}{5!}\right)2^4\cdot q2q2+\left(\frac{-6}{7!}\right)2^2\cdot q2=0$$
$$\left(\frac{1}{1!}\right)2^8\cdot q4q4+\left(\frac{1}{3!}\right)2^6\cdot q2q4+\left(\frac{-4}{5!}\right)2^4\cdot q4=0$$
$$\left(\frac{1}{1!}\right)2^8\cdot q2q6+\left(\frac{-2}{3!}\right)2^6\cdot q6=0$$

We add all terms in the triangle and get one long sum, and this must be zero.

$$\frac{\left(2^8 \cdot q6q2 + 2^8 \cdot q4q4 + 2^8 \cdot q2q6\right)}{1!} + \frac{\left(2^6 \cdot q4q2 + 2^6 \cdot q2q4\right) - 2 \cdot 2^6 \cdot q6}{3!}$$
$$+ \frac{\left(2^4 \cdot q2q2\right) - 4 \cdot 2^4 \cdot q4}{5!} + \frac{-6 \cdot 2^2 \cdot q2}{7!} = 0$$

From an earlier work I have, that

$$-9q8 = q6q2 + q4q4 + q2q6$$

$$-7q6 = q4q2 + q4q2$$

$$-5q4 = q2q2$$

Then it must be true that

$$-9q8 \cdot 2^8 = q6q2 \cdot 2^8 + q4q4 \cdot 2^8 + q2q6 \cdot 2^8$$

$$-7q6 \cdot 2^6 = q4q2 \cdot 2^6 + q4q2 \cdot 2^6$$

$$-5q4 \cdot 2^4 = q2q2 \cdot 2^4$$

We replace the right side with the left in the long equation and get:

$$\frac{\left(-9 \cdot 2^8 \cdot q8\right)}{1!} + \frac{\left(-7 \cdot 2^6 \cdot q6\right) - 2 \cdot 2^6 \cdot q6}{3!} + \frac{\left(-5 \cdot 2^4 \cdot q4\right) - 4 \cdot 2^4 \cdot q4}{5!} + \frac{-6 \cdot 2^2 \cdot q2}{7!} = 0$$

and:

$$\frac{-9 \cdot 2^8 \cdot q8}{1!} + \frac{-9 \cdot 2^6 \cdot q6}{3!} + \frac{-9 \cdot 2^4 \cdot q4}{5!} + \frac{-6 \cdot 2^2 \cdot q2}{7!} = 0$$

With a homage to the symmetri we could add:

$$\frac{-3 \cdot 2^2 \cdot q2}{7!} + \frac{9 \cdot 2^1 \cdot q1}{8!} + \frac{-9 \cdot 2^0 \cdot q0}{9!} = 0$$

That is a rather cheap maneuvre, because the 3 terms are allways zero, as I will demonstrate below.
We get:

$$\frac{-9 \cdot 2^8 \cdot q8}{1!} + \frac{-9 \cdot 2^6 \cdot q6}{3!} + \frac{-9 \cdot 2^4 \cdot q4}{5!} + \frac{-9 \cdot 2^2 \cdot q2}{7!} + \frac{9 \cdot 2^1 \cdot q1}{8!} + \frac{-9 \cdot 2^0 \cdot q0}{9!} = 0$$

We can divide on both side of the equatiom with -9, and get:

$$\frac{1}{1!} \cdot 2^8 \cdot q8 + \frac{1}{3!} \cdot 2^6 \cdot q6 + \frac{1}{5!} \cdot 2^4 \cdot q4 + \frac{1}{7!} \cdot 2^2 \cdot q2 + \frac{-1}{8!} \cdot 2^1 \cdot q1 + \frac{1}{9!} \cdot 2^0 \cdot q0 = 0$$

and this is the next mirrorequation nr. 1. And it can be used to construct the next mirrorequation nr. 1, starting with $\frac{1}{1!} \cdot 2^{10} \cdot q10$.

The following equation is in an earlier work called **the basic equation** to calculate $q8$. $q8$ is simply the rest of the equation with opposite sign.

$$\frac{1}{1!}\cdot q8+\frac{1}{3!}\cdot q6+\frac{1}{5!}\cdot q4+\frac{1}{7!}\cdot q2+\frac{-1}{8!}\cdot q1+\frac{1}{9!}\cdot q0=0$$

If we subtract this equation from the mirrorequation above, we will get the mirrorequation nr. 1 in a form more relevant for this work.

$$\frac{1}{1!}\cdot(2^8-1)q8+\frac{1}{3!}\cdot(2^6-1)q6+\frac{1}{5!}\cdot(2^4-1)q4+\frac{1}{7!}\cdot(2^2-1)q2+\frac{-1}{8!}\cdot(2^1-1)q1+\frac{1}{9!}\cdot(2^0-1)q0=0$$

Note the $q0$-term gets zero.

We can keep on in this way adding more and more ever longer equations, multiplying them with $2^2\cdot q2$, $2^4\cdot q4$ and so on until $2^{2\cdot n-2}\cdot q(2\cdot n-2)$.
We will end up with $(n-1)$ equations in the triangle and get this equation:

$$\frac{1}{1!}\cdot(2^{2\cdot n}-1)q(2\cdot n)+\frac{1}{3!}\cdot(2^{n-2}-1)q(2\cdot n-2)+\cdots+\frac{1}{(2\cdot n-1)!}\cdot(2^2-1)q2+\frac{-1}{(2\cdot n)!}\cdot(2^1-1)q1=0$$

Now this work it's about Genocchi's numbers not so much about the q-values.

$$g(2\cdot n)=2\cdot(2\cdot n)!\cdot(2^{2\cdot n}-1)\cdot q(2\cdot n)$$

$$\frac{1}{1!}\cdot(2^8-1)q8+\frac{1}{3!}\cdot(2^6-1)q6+\frac{1}{5!}\cdot(2^4-1)q4+\frac{1}{7!}\cdot(2^2-1)q2+\frac{-1}{8!}\cdot(2^1-1)q1=0$$

The new mirrorequation nr. 1 above can be written in this way:

$$\frac{9!}{1!8!}\cdot2\cdot8!(2^8-1)q8+\frac{9!}{3!6!}\cdot2\cdot6!(2^6-1)q6+\frac{9!}{5!4!}\cdot2\cdot4!(2^4-1)q4+\frac{9!}{7!2!}\cdot2\cdot2!(2^2-1)q2+\frac{-9!}{8!1!}\cdot2\cdot1!(2^1-1)q1=0$$

We now insert Genocchi's first 3 numbers here:
$g2=2\cdot2!(2^2-1)q2,\ g4=2\cdot4!(2^4-1)q4,\ g6=2\cdot6!(2^6-1)q6$ og $g8=2\cdot8!(2^8-1)q8$
and remembering $q1=\frac{1}{2}$
$\frac{-9!}{8!\cdot1!}\cdot2\cdot1!(2^1-1)\cdot q1=-9,$
We get:

$$\frac{9!}{1!\cdot8!}\cdot g8+\frac{9!}{3!\cdot6!}\cdot g6+\frac{9!}{5!\cdot4!}\cdot g4+\frac{9!}{7!\cdot2!}\cdot g2+-9=0$$

And have now got an algoritm (recursiv) to express and to calculate the Genocchi numbers from all earlier calculated ones.

$$9\cdot g8=-\left(\frac{9!}{3!\cdot6!}\cdot g6+\frac{9!}{5!\cdot4!}\cdot g4+\frac{9!}{7!\cdot2!}\cdot g2+-9\right)$$

Is $9\cdot g8$ a whole number?
Yes, it has to be. In the end of this demonstration I will demonstrate, that fractions in this form

$$\frac{n!}{k!\cdot(n-k)!}$$

are always whole numbers.
We can call them **Pascal's coefficients**.
I have calculated g2, g4 og g6 to 1, -1 og 3.
All the terms are products of whole numbers. The sum must be whole.
And what do our n. mirrorequation nr. 1 looks like, when we replace all our q-values by Genocchi numbers?

$$\frac{(2\cdot n+1)!}{1!(2n)!}\cdot g(2\cdot n)+\frac{(2\cdot n+1)!}{3!(2n-2)!}\cdot g(2\cdot n-2)+\cdots+\frac{(2\cdot n+1)!}{(2n-3)!4!}\cdot g4+\frac{(2\cdot n+1)!}{(2n-1)!2!}\cdot g2+-\frac{(2\cdot n+1)!}{(2n)!1!}=0$$

or

$$(2\cdot n+1)\cdot g(2\cdot n)=-\left(\frac{(2\cdot n+1)!}{3!(2n-2)!}\cdot g(2\cdot n-2)+\cdots+\frac{(2\cdot n+1)!}{(2n-3)!4!}\cdot g4+\frac{(2\cdot n+1)!}{(2n-1)!2!}\cdot g2+-(2\cdot n+1)\right)$$

Is $(2\cdot n+1)\cdot g(2\cdot n)$ now a whole number?
Yes, it ought to, the fractions are **Pascal's coefficients**.
When you add the 2 numbers before the exclamation marks: ! in the denominator, you get the number in the numerator before that same mark ! and Genocchi's numbers smaller than $g(2\cdot n)$ are supposed to be known.
Products of whole numbers add up to one whole number.

Insertion

$$\frac{-3\cdot 2^2\cdot q2}{7!}+\frac{9\cdot 2^1\cdot q1}{8!}+\frac{-9\cdot 2^0\cdot q0}{9!}=0$$

I promised above to demonstrate that the 3 last terms in the mirrorequation nr. 1 add up to zero.

We replace 9 by $(2\cdot n+1)$ and 8 by $(2\cdot n)$ and 7 by $(2\cdot n-1)$
and get:

$$\frac{-3\cdot 2^2\cdot q2}{(2\cdot n-1)!}+\frac{(2\cdot n+1)\cdot 2^1\cdot q1}{(2\cdot n)!}+\frac{-(2\cdot n+1)\cdot 2^0\cdot q0}{(2\cdot n+1)!}=0$$

$$\frac{-3\cdot 2^2\cdot q2}{(2\cdot n-1)!}+\frac{(2\cdot n+1)\cdot 2^1\cdot q1}{(2\cdot n)!}+\frac{-1\cdot 2^0\cdot q0}{(2\cdot n)!}=0$$

$q2=\frac{1}{12}$ og $q1=\frac{1}{2}$ og $q0=1$ implies

$$\frac{-1}{(2\cdot n-1)!}+\frac{(2\cdot n+1)}{(2\cdot n)!}+\frac{-1\cdot 2^0\cdot q0}{(2\cdot n)!}=0$$

$$\frac{-1}{(2\cdot n-1)!}+\frac{(2\cdot n+1)}{(2\cdot n)!}+\frac{-1}{(2\cdot n)!}=0$$

$$\frac{-1}{(2\cdot n-1)!}+\frac{(2\cdot n)}{(2\cdot n)!}=0$$

$$\frac{-1}{(2\cdot n-1)!}+\frac{1}{(2\cdot n-1)!}=0$$

Q.E.D.

end of insertion

1.1.4 A new algoritm to calculate $q(2n)\cdot(2^{2n}-1)$ from all earlier q-values of the whole numbers

Below we have 3 **mirrorequation nr. 1**, we now know the right sides are always zero.

$$\left(\frac{1}{1!}\right)q6(2^6-1)+\left(\frac{1}{3!}\right)q4(2^4-1)+\left(\frac{1}{5!}\right)q2(2^2-1)+\left(\frac{-1}{6!}\right)q1(2^1-1)+\left(\frac{1}{7!}\right)q0(2^0-1)=0$$
$$\left(\frac{1}{1!}\right)q4(2^4-1)+\left(\frac{1}{3!}\right)q2(2^2-1)+\left(\frac{-1}{4!}\right)q1(2^1-1)+\left(\frac{1}{5!}\right)q0(2^0-1)=0$$
$$\left(\frac{1}{1!}\right)q2(2^2-1)+\left(\frac{-1}{2!}\right)q1(2^1-1)+\left(\frac{1}{3!}\right)q0(2^0-1)=0$$

I multiply the top equation with $q2$, the middle with $q4$ and the last with $q6$.

$$\left(\frac{1}{1!}\right)q2q6(2^6-1)+\left(\frac{1}{3!}\right)q2q4(2^4-1)+\left(\frac{1}{5!}\right)q2q2(2^2-1)+\left(\frac{-1}{6!}\right)q2q1(2^1-1)+\left(\frac{1}{7!}\right)q2q0(2^0-1)=0$$
$$\left(\frac{1}{1!}\right)q4q4(2^4-1)+\left(\frac{1}{3!}\right)q4q2(2^2-1)+\left(\frac{-1}{4!}\right)q4q1(2^1-1)+\left(\frac{1}{5!}\right)q4q0(2^0-1)=0$$
$$\left(\frac{1}{1!}\right)q6q2(2^2-1)+\left(\frac{-1}{2!}\right)q6q1(2^1-1)+\left(\frac{1}{3!}\right)q6q0(2^0-1)=0$$

We can remove the $q0$-terms. They are zero.

$$\left(\frac{1}{1!}\right)q2q6(2^6-1)+\left(\frac{1}{3!}\right)q2q4(2^4-1)+\left(\frac{1}{5!}\right)q2q2(2^2-1)+\left(\frac{-1}{6!}\right)q2q1(2^1-1)=0$$
$$\left(\frac{1}{1!}\right)q4q4(2^4-1)+\left(\frac{1}{3!}\right)q4q2(2^2-1)+\left(\frac{-1}{4!}\right)q4q1(2^1-1))=0$$
$$\left(\frac{1}{1!}\right)q6q2(2^2-1)+\left(\frac{-1}{2!}\right)q6q1(2^1-1)=0$$

We add vertical and and place the $q1$-terms at last. The whole thing must be zero.

$$\left(\frac{1}{1!}\right)q2q6(2^2-1)+\left(\frac{1}{1!}\right)q4q4(2^4-1)+\left(\frac{1}{1!}\right)q6q2(2^6-1)+\left(\frac{1}{3!}\right)q2q4(2^2-1)+\left(\frac{1}{3!}\right)q4q2(2^4-1)$$
$$+\left(\frac{1}{5!}\right)q2q2(2^2-1)-\left(\frac{1}{2!}\right)q1q6(2^1-1)-\left(\frac{1}{4!}\right)q1q4(2^1-1)-\left(\frac{1}{6!}\right)q1q2(2^1-1)=0$$

I intend to demonstrate below that

$$-\left(\frac{1}{2!}\right)q1q6(2^1-1)-\left(\frac{1}{4!}\right)q1q4(2^1-1)-\left(\frac{1}{6!}\right)q1q2(2^1-1)=-\left(\frac{1}{7!}\right)q1q1(2^1-1)+\left(\frac{1}{8!}\right)q1q0(2^1-1)$$

So we can replace the expression before the sign of equation with that after in the long sum.
We now have:

$$\left(\frac{1}{1!}\right)q2q6(2^2-1)+\left(\frac{1}{1!}\right)q4q4(2^4-1)+\left(\frac{1}{1!}\right)q6q2(2^6-1)+\left(\frac{1}{3!}\right)q2q4(2^2-1)+\left(\frac{1}{3!}\right)q4q2(2^4-1)$$

$$+\left(\frac{1}{5!}\right)q2q2(2^2-1)-\left(\frac{1}{7!}\right)q1q1(2^1-1)+\left(\frac{1}{8!}\right)q1q0(2^1-1)$$

Remembering $q0=1$, $q1=\frac{1}{2}$ and $q2=\frac{1}{12}$ it is obvious that this will always be true

$$-\left(\frac{1}{(2\cdot n-1)!}\right)q1q1(2^1-1)+\left(\frac{1}{2\cdot n!}\right)q1q0(2^1-1)=-\left(\frac{1}{(2\cdot n-1)!}\right)q2(2^2-1)+\left(\frac{1}{(2\cdot n)!}\right)q1(2^1-1)$$

in all future equations.
For $2\cdot n=8$ we can replace

$$-\left(\frac{1}{7!}\right)q1q1(2^1-1)+\left(\frac{1}{8!}\right)q1q0(2^1-1)$$

with

$$-\left(\frac{1}{7!}\right)q2(2^2-1)+\left(\frac{1}{8!}\right)q1(2^1-1)$$

We now have:

$$\left(\frac{1}{1!}\right)q2q6(2^2-1)+\left(\frac{1}{1!}\right)q4q4(2^4-1)+\left(\frac{1}{1!}\right)q6q2(2^6-1)+\left(\frac{1}{3!}\right)q2q4(2^2-1)+\left(\frac{1}{3!}\right)q4q2(2^4-1)$$

$$+\left(\frac{1}{5!}\right)q2q2(2^2-1)-\left(\frac{1}{7!}\right)q2(2^2-1)+\left(\frac{1}{8!}\right)q1(2^1-1)=0$$

We can gather the $\frac{1}{1!}$-terms, the $\frac{1}{3!}$-terms and so on in parentheses
We get:

$$\frac{1}{1!}\cdot\left(q2q6(2^2-1)+q4q4(2^4-1)+q6q2(2^6-1)\right)+\frac{1}{3!}\cdot\left(q2q4(2^2-1)+q4q2(2^4-1)\right)$$

$$+\left(\frac{1}{5!}\right)q2q2(2^2-1)-\left(\frac{1}{7!}\right)q2(2^2-1)+\left(\frac{1}{8!}\right)q1(2^1-1)=0$$

I now suggest a rule

$$\begin{aligned}&-q(2n)\cdot(2^{2n}-1)=\\&q2\cdot q(2n-2)\cdot(2^2-1)+q(4)\cdot q(2n-4)\cdot(2^4-1)\quad+\dots+q(2n-4)\cdot q(4)\cdot(2^{2n-4}-1)\\&+q(2n-2)\cdot q(2)\cdot(2^{2n-2}-1)\end{aligned}$$

You can express it in this way, a new term $q(2n)\cdot(2^{2n}-1)$ can be calculated from the sum of all the products of pairs of earlier q-values $(q2t)\cdot q(2n-2t)$ the sum of $2t+(2n-2t)$ being $(2n)$, an example: $q4\cdot q(2n-4)$.
Note we find most of the pairs 2 times, example: $q4\cdot q(2n-4)$ and later $q(2n-4)\cdot q(4)$.
But the term $qn\cdot qn$ occur only 1 time in the middle of the equation, and only if n is even.
We shall name these terms **middle terms**. Example: $q4\cdot q4$

These products are then multiplied with parentheses containing a power of 2 and -1
The exponents of the powers of 2 increases with 2 from 2^2 to 2^{2n-2}.
This suggestion implies then,
that for $2n = 4$ is

$$-q(2n) \cdot (2^{2n} - 1) = -q4 \cdot (2^4 - 1) = q2 \cdot q2 \cdot (2^2 - 1)$$

and for $2n = 6$ is

$$-q(2n) \cdot (2^{2n} - 1) = -q6 \cdot (2^6 - 1) = q2 \cdot q4 \cdot (2^2 - 1) + q4 \cdot q2 \cdot (2^4 - 1)$$

We will place these shorter terms $-q4 \cdot (2^4 - 1)$ and $-q6 \cdot (2^6 - 1)$ in our long equation

$$\frac{1}{1!} \cdot \Big(q2q6(2^2 - 1) + q4q4(2^4 - 1) + q6q2(2^6 - 1)\Big) + \frac{1}{3!} \cdot \Big(q2q4(2^2 - 1) + q4q2(2^4 - 1)\Big)$$

$$+ \left(\frac{1}{5!}\right) q2q2(2^2 - 1) - \left(\frac{1}{7!}\right) q2(2^2 - 1) + \left(\frac{1}{8!}\right) q1(2^1 - 1) = 0$$

and get:

$$\left(\frac{1}{1!}\right) \cdot \Big(q2q6(2^2 - 1) + q4q4 \cdot (2^4 - 1) + q6q2 \cdot (2^6 - 1)\Big) - \left(\frac{1}{3!}\right) \cdot q6 \cdot (2^6 - 1)$$

$$- \left(\frac{1}{5!}\right) q4 \cdot (2^4 - 1) - \left(\frac{1}{7!}\right) q2 \cdot (2^2 - 1) + \left(\frac{1}{8!}\right) q1 \cdot (2^1 - 1) = 0$$

or

$$\left(\frac{1}{1!}\right) \cdot \Big(q2q6 \cdot (2^2 - 1) + q4q4 \cdot (2^4 - 1) + q6q2 \cdot (2^6 - 1)\Big)$$

$$- \left(\frac{1}{3!}\right) \cdot q6 \cdot (2^6 - 1) - \left(\frac{1}{5!}\right) q4 \cdot (2^4 - 1) - \left(\frac{1}{7!}\right) q2 \cdot (2^2 - 1) + \left(\frac{1}{8!}\right) q1 \cdot (2^1 - 1) = 0$$

And now we should discover, that our long equaqtion is rearly **the mirrorequation nr. 1** multiplied with -1:

$$-\left(\frac{1}{1!}\right) \cdot q8 \cdot (2^8 - 1) - \left(\frac{1}{3!}\right) \cdot q6 \cdot (2^6 - 1) - \left(\frac{1}{5!}\right) q4 \cdot (2^4 - 1) - \left(\frac{1}{7!}\right) q2 \cdot (2^2 - 1) + \left(\frac{1}{8!}\right) q1 \cdot (2^1 - 1) = 0$$

Both equations are zero. Then it must be true, that

$$\left(\frac{1}{1!}\right) \cdot \Big(q2q6(2^2 - 1) + q4q4(2^4 - 1) + q6q2(2^6 - 1)\Big) = -\left(\frac{1}{1!}\right) \cdot q8 \cdot (2^8 - 1)$$

Is our rule for $2n = 8$ in accordance with the parenthesis in the beginning of the equation?

$$-q(2n) \cdot (2^{2n} - 1) = -q8 \cdot (2^8 - 1) = q2 \cdot q6 \cdot (2^2 - 1) + q4 \cdot q4 \cdot (2^4 - 1) + q6 \cdot q2 \cdot (2^6 - 1)$$

Yes, the (2n)'s in the small products are 2+6=8, 4+4=8 og 6+2=8, and the exponents of the powers of 2 in parentheses increase from 2 to 4 to 6.
Note **middle terms** $q4q4(2^4-1)$.

Insertion begin
I promised above to demonstrate, that

$$-\left(\frac{1}{2!}\right)q1q6(2^1-1)-\left(\frac{1}{4!}\right)q1q4(2^1-1)-\left(\frac{1}{6!}\right)q1q2(2^1-1)=-\left(\frac{1}{7!}\right)q1q1(2^1-1)+\left(\frac{1}{8!}\right)q1q0(2^1-1)$$

is true, but that is the same as to prove that

$$-\left(\frac{1}{2!}\right)q1q6(2^1-1)-\left(\frac{1}{4!}\right)q1q4(2^1-1)-\left(\frac{1}{6!}\right)q1q2(2^1-1)+\left(\frac{1}{7!}\right)q1q1(2^1-1)-\left(\frac{1}{8!}\right)q1q0(2^1-1)=0$$

The parentheses are 1, and we can divide on both side of the sign of equation with $-q1$, and we end up with having to prove this equation is correct.

$$\left(\frac{1}{2!}\right)q6+\left(\frac{1}{4!}\right)q4+\left(\frac{1}{6!}\right)q2-\left(\frac{1}{7!}\right)q1+\left(\frac{1}{8!}\right)q0=0$$

We fetch **the basic equation** defining $q8$.

$$\left(\frac{1}{1!}\right)q8+\left(\frac{1}{3!}\right)q6+\left(\frac{1}{5!}\right)q4+\left(\frac{1}{7!}\right)q2-\left(\frac{1}{8!}\right)q1+\left(\frac{1}{9!}\right)q0=0$$

We multiply on both side of the sign of equation with 9

$$\left(\frac{9}{1!}\right)q8+\left(\frac{9}{3!}\right)q6+\left(\frac{9}{5!}\right)q4+\left(\frac{9}{7!}\right)q2-\left(\frac{9}{8!}\right)q1+\left(\frac{9}{9!}\right)q0=0$$

We divide the equation in 2 parts. They must be numerical the same but with opposite sign.
1. part: **my equation of the increasing numerator** which lack a $q8$. There should have been 10 not 9 $q8$, and therefore the left of the equation must be $-q8$ instead of zero.

$$\left(\frac{9}{1!}\right)q8+\left(\frac{7}{3!}\right)q6+\left(\frac{5}{5!}\right)q4+\left(\frac{3}{7!}\right)q2-\left(\frac{2}{8!}\right)q1+\left(\frac{1}{9!}\right)q0=-\left(\frac{1}{1!}\right)q8$$

2. part has then to be $q8$ (the 2 parts togther being zero):

$$\left(\frac{2}{3!}\right)q6+\left(\frac{4}{5!}\right)q4+\left(\frac{6}{7!}\right)q2-\left(\frac{7}{8!}\right)q1+\left(\frac{8}{9!}\right)q0=\left(\frac{1}{1!}\right)q8$$

Wir now add **the basic equation** defining $q8$, and place the $q8$ on the right side as $-q8$

$$\left(\frac{1}{3!}\right)q6+\left(\frac{1}{5!}\right)q4+\left(\frac{1}{7!}\right)q2-\left(\frac{1}{8!}\right)q1+\left(\frac{1}{9!}\right)q0=-\left(\frac{1}{1!}\right)q8$$

and get

$$\left(\frac{3}{3!}\right) q6 + \left(\frac{5}{5!}\right) q4 + \left(\frac{7}{7!}\right) q2 - \left(\frac{8}{8!}\right) q1 + \left(\frac{9}{9!}\right) q0 = 0$$

We reduce the fractions, and get

$$\left(\frac{1}{2!}\right) q6 + \left(\frac{1}{4!}\right) q4 + \left(\frac{1}{6!}\right) q2 - \left(\frac{1}{7!}\right) q1 + \left(\frac{1}{8!}\right) q0 = 0$$

Q.E.D.
Insertion end

1.1.5 The equation of the decreasing numerators with even numerators in all fractions eccept the first and the last.

With an allusion to the insertion above, I say: you fetch a **a mirrorequation nr. 1** and multiply it on both side of the sign of equation with 9.

$$\frac{9}{1!} \cdot (2^8 - 1)q8 + \frac{9}{3!} \cdot (2^6 - 1)q6 + \frac{9}{5!} \cdot (2^4 - 1)q4 + \frac{9}{7!} \cdot (2^2 - 1)q2 + \frac{-9}{8!} \cdot (2^1 - 1)q1 + \frac{9}{9!} \cdot (2^0 - 1)q0 = 0$$

As in the insertion above we divide the equation in 2 parts. Normally the 2 parts will be numerical the same but with opposite sign.
But if we are lucky, they will be quite the same, and have the same sign too. Then both equation have zero on the right side of the sign of eaquation.

$$\frac{8}{1!} \cdot (2^8 - 1)q8 + \frac{7}{3!} \cdot (2^6 - 1)q6 + \frac{5}{5!} \cdot (2^4 - 1)q4 + \frac{3}{7!} \cdot (2^2 - 1)q2 + \frac{-2}{8!} \cdot (2^1 - 1)q1 <?> 0$$

$$\frac{1}{1!} \cdot (2^8 - 1)q8 + \frac{2}{3!} \cdot (2^6 - 1)q6 + \frac{4}{5!} \cdot (2^4 - 1)q4 + \frac{6}{7!} \cdot (2^2 - 1)q2 + \frac{-7}{8!} \cdot (2^1 - 1)q1 = q1 <?> 0$$

Above we see such a lucky dividing of a **mirrorequation nr. 1**
We will concentrate on the 2. part. It has decreasing numerators (decreasing with increasing $2n$ in $q(2n)$) and even numerators in all fractions eccept the first and the last. It is the 4. equation of this type. And we will demonstrate, it is correct (in fact equals zero), when the 3 first equations of this type is zero.

Below we have the 3 first **equations of the decreasing numerators with even numerators in all fractions eccept the first and the last** and calculated being zero.

$$\left(\frac{1}{1!}\right) q6(2^6 - 1) + \left(\frac{2}{3!}\right) q4(2^4 - 1) + \left(\frac{4}{5!}\right) q2(2^2 - 1) + \left(\frac{-5}{6!}\right) q1(2^1 - 1) = 0$$

$$\left(\frac{1}{1!}\right) q4(2^4 - 1) + \left(\frac{2}{3!}\right) q2(2^2 - 1) + \left(\frac{-3}{4!}\right) q1(2^1 - 1) = 0$$

$$\left(\frac{1}{1!}\right) q2(2^2 - 1) + \left(\frac{-1}{2!}\right) q1(2^1 - 1) = 0$$

The first we multiply with $q2$, the middle with $q4$ and the last with $q6$.

$$\left(\frac{1}{1!}\right) q2q6(2^6-1) + \left(\frac{2}{3!}\right) q2q4(2^4-1) + \left(\frac{4}{5!}\right) q2q2(2^2-1) + \left(\frac{-5}{6!}\right) q2q1(2^1-1) = 0$$
$$\left(\frac{1}{1!}\right) q4q4(2^4-1) + \left(\frac{2}{3!}\right) q4q2(2^2-1) + \left(\frac{-3}{4!}\right) q4q1(2^1-1) = 0$$
$$\left(\frac{1}{1!}\right) q6q2(2^2-1) + \left(\frac{-1}{2!}\right) q6q1(2^1-1) = 0$$

We add vertical and and place the $q1$-terms at last. The whole thing must be zero.

$$\left(\frac{1}{1!}\right) q2q6(2^6-1) + \left(\frac{1}{1!}\right) q4q4(2^4-1) + \left(\frac{1}{1!}\right) q6q2(2^2-1)$$
$$+ \left(\frac{2}{3!}\right) q2q4(2^2-1) + \left(\frac{2}{3!}\right) q4q2(2^4-1)$$
$$+ \left(\frac{4}{5!}\right) q2q2(2^2-1)$$
$$- \left(\frac{1}{2!}\right) q1q6(2^1-1) - \left(\frac{3}{4!}\right) q1q4(2^1-1) - \left(\frac{5}{6!}\right) q1q2(2^1-1) = 0$$

In the section above about a new algoritm to calculate $q(2n)\cdot(2^n-1)$ from all earlier q-values of the even numbers, we learnt,
that for $2n=4$ is

$$-q(2n)\cdot(2^{2n}-1) = -q4\cdot(2^4-1) = q2\cdot q2\cdot(2^2-1)$$

and for $2n=6$ is

$$-q(2n)\cdot(2^{2n}-1) = -q6\cdot(2^6-1) = q2\cdot q4\cdot(2^2-1) + q4\cdot q2\cdot(2^4-1)$$

and for $2n=8$ is

$$-q(2n)\cdot(2^{2n}-1) = -q8\cdot(2^8-1) = q6\cdot q2\cdot(2^2-1) + q4\cdot q4\cdot(2^4-1) + q2\cdot q6\cdot(2^6-1)$$

We insert these shorter terms in the long equation and get:

$$- \left(\frac{1}{1!}\right) q8\cdot(2^8-1) - \left(\frac{2}{3!}\right) q6\cdot(2^6-1) - \left(\frac{4}{5!}\right) q4\cdot(2^4-1)$$
$$- \left(\frac{1}{2!}\right) q1q6(2^1-1) - \left(\frac{3}{4!}\right) q1q4(2^1-1) - \left(\frac{5}{6!}\right) q1q2(2^1-1) = 0$$

We can multiply the equation with -1 and get:

$$\left(\frac{1}{1!}\right) q8\cdot(2^8-1) + \left(\frac{2}{3!}\right) q6\cdot(2^6-1) + \left(\frac{4}{5!}\right) q4\cdot(2^4-1)$$

$$+\left(\frac{1}{2!}\right)q1q6(2^1-1)+\left(\frac{3}{4!}\right)q1q4(2^1-1)+\left(\frac{5}{6!}\right)q1q2(2^1-1)=0$$

We now define a and b:

$$a:=\left(\frac{1}{2!}\right)q1q6(2^1-1)+\left(\frac{3}{4!}\right)q1q4(2^1-1)+\left(\frac{5}{6!}\right)q1q2(2^1-1))-\left(\frac{6}{7!}\right)q1q1(2^1-1))+\left(\frac{7}{8!}\right)q1q0(2^1-1)$$

$$b:=\left(\frac{1}{2!}\right)q1q6(2^1-1)+\left(\frac{1}{4!}\right)q1q4(2^1-1)+\left(\frac{1}{6!}\right)q1q2(2^1-1))-\left(\frac{1}{7!}\right)q1q1(2^1-1))+\left(\frac{1}{8!}\right)q1q0(2^1-1)$$

In **Insertion** above we proved $b=0$.

We will now demonstrate, that a + b, make the **basic equation** for calculating $q6$.

The basic equations are zero.

$$a{+}b=\left(\frac{2}{2!}\right)q1q6(2^1-1)+\left(\frac{4}{4!}\right)q1q4(2^1-1)+\left(\frac{6}{6!}\right)q1q2(2^1-1))-\left(\frac{7}{7!}\right)q1q1(2^1-1))+\left(\frac{8}{8!}\right)q1q0(2^1-1)$$

We reduce the fractions and divide all terms with $q1$ and $(2^1-1)=1$.

$$a+b=\left(\frac{1}{1!}\right)q6+\left(\frac{1}{3!}\right)q4+\left(\frac{1}{5!}\right)q2-\left(\frac{1}{6!}\right)q1+\left(\frac{1}{7!}\right)q0=0$$

As promised we get the **basic equation** for calculating $q6$, which is zero.
a+b is zero, and b is zero, implies, a must be zero.

Q.E.D.

$$a:=\left(\frac{1}{2!}\right)q1q6(2^1-1)+\left(\frac{3}{4!}\right)q1q4(2^1-1)+\left(\frac{5}{6!}\right)q1q2(2^1-1))-\left(\frac{6}{7!}\right)q1q1(2^1-1))+\left(\frac{7}{8!}\right)q1q0(2^1-1)=0$$

a=0 implies, that

$$\left(\frac{1}{2!}\right)q1q6(2^1-1)+\left(\frac{3}{4!}\right)q1q4(2^1-1)+\left(\frac{5}{6!}\right)q1q2(2^1-1))=+\left(\frac{6}{7!}\right)q1q1(2^1-1))-\left(\frac{7}{8!}\right)q1q0(2^1-1)$$

We can replace the 3 terms on the left side of the sign of equation with the 2 terms on the right in the long equation and get:

$$\left(\frac{1}{1!}\right)q8\cdot(2^8-1)+\left(\frac{2}{3!}\right)q6\cdot(2^6-1)+\left(\frac{4}{5!}\right)q4\cdot(2^4-1)+\left(\frac{6}{7!}\right)q1q1(2^1-1))-\left(\frac{7}{8!}\right)q1q0(2^1-1)$$

Remembering $q0=1$, $q1=\frac{1}{2}$ and $q2=\frac{1}{12}$, we have:

$$\left(\frac{6}{7!}\right)q1q1(2^1-1)=\left(\frac{6}{7!}\right)q2(2^2-1)$$

.
We insert this in the long equation and remove $q0$ (being 1) in the last term, and we get:

$$\left(\frac{1}{1!}\right) q8\cdot(2^8-1)+\left(\frac{2}{3!}\right) q6\cdot(2^6-1)+\left(\frac{4}{5!}\right) q4\cdot(2^4-1)+\left(\frac{6}{7!}\right) q2\cdot(2^2-1))-\left(\frac{7}{8!}\right)\cdot q1\cdot(2^1-1)=0$$

And there at last we have **the 4. equation of the decreasing numerators with even numerators in all fractions eccept the first and the last.** We could now with 4 equations construct the 5. equation starting with $q10\cdot(2^{10}-1)$ and so on.

If we multiply all terms in our new equation of the decreasing numerators with $9!$, and insert the first 4 of Genocchi's numbers here: $g2=2\cdot 2!(2^2-1)q2$, $g4=2\cdot 4!(2^4-1)q4$, $g6=2\cdot 6!(2^6-1)q6$ og $g8=2\cdot 8!(2^8-1)q8$ in the equation and $\frac{-9!}{8!\cdot 1!}\cdot 2\cdot 1!(2^1-1)\cdot q1=-9$, because $q1=\frac{1}{2}$ we get:

$$\frac{9!}{1!\cdot 8!}\cdot g8+\frac{9!}{3!\cdot 6!}\cdot 2\cdot g6+\frac{9!}{5!\cdot 4!}\cdot 4\cdot g4+\frac{9!}{7!\cdot 2!}\cdot 6\cdot g2+-9\cdot 7=0$$

We can now again express $9\cdot g8$ by all earlier calculated Genocchi-numbers:

$$9\cdot g8=-\left(\frac{9!}{3!\cdot 6!}\cdot 2\cdot g6+\frac{9!}{5!\cdot 4!}\cdot 4\cdot g4+\frac{9!}{7!\cdot 2!}\cdot 6\cdot g2+-9\cdot 7\right)$$

Is $9\cdot q8$ a whole number?
It should be, the fractions are our **Pascal-coefficients**, (when you add the 2 numbers before the exclamation marks: ! in the denominator, you get the number in the numerator before that same mark !)
I will later return to these Pascal-coefficients. But they are whole numbers all right.
I have calculated g2, g4 og g6 to 1, -1 og 3.

$$\frac{11!}{1!\cdot 10!}\cdot g10+\frac{11!}{3!\cdot 8!}\cdot 2\cdot g8+\frac{11!}{5!\cdot 6!}\cdot 4\cdot g6+\frac{11!}{7!\cdot 4!}\cdot 6\cdot g4+\frac{11!}{9!\cdot 2!}\cdot 8\cdot g2+-11\cdot 9=0$$

$$11\cdot g10=\frac{11!}{3!\cdot 8!}\cdot 2\cdot g8+\frac{11!}{5!\cdot 6!}\cdot 4\cdot g6+\frac{11!}{7!\cdot 4!}\cdot 6\cdot g4+\frac{11!}{9!\cdot 2!}\cdot 8\cdot g2+-11\cdot 9=0$$

And what does our **n. equation of the decreasing numerators with even numerators in all fractions eccept the first and the last** look like, when we replace the q-values with Genocchi numbers?

$$\frac{(2n+1)!}{1!(2n)!}\cdot g(2n)+\frac{(2n+1)!}{3!(2n-2)!}2g(2n-2)+\cdots+\frac{(2n+1)!}{(2n-3)!4!}(2n-4)g4+\frac{(2n+1)!}{(2n-1)!2!}(2n-2)g2$$

$$-(2n+1)(2n-1)=0$$

or

$$(2{\cdot}n{+}1){\cdot}g(2{\cdot}n) = -\left(\frac{(2\cdot n+1)!}{3!(2n-2)!}2g(2n-2)+\cdots+\frac{(2\cdot n+1)!}{(2n-3)!4!}(2n-4)g4+\frac{(2\cdot n+1)!}{(2n-1)!2!}(2n-2)g2\right)$$

$$+(2n+1)(2n-1)$$

Is $(2n+1)\cdot g(2n)$ a whole number?
Yes, the fractions are **Pascal's coefficients**. and it is assumed we have found all earlier Genocchi number whole.

1.1.6 My mirrorequation nr. 2

If we add our new constructed **equation of the decreasing numerators with even numerators in all fractions eccept in the first and the last term** and the corresponding **mirrorequation nr. 1**, we get **the mirrorequation nr. 2**.

$$\left(\frac{1}{1!}\right)q8{\cdot}(2^8-1)+\left(\frac{2}{3!}\right)q6{\cdot}(2^6-1)+\left(\frac{4}{5!}\right)q4{\cdot}(2^4-1)+\left(\frac{6}{7!}\right)q2{\cdot}(2^2-1))-\left(\frac{7}{8!}\right){\cdot}q1{\cdot}(2^1-1)=0$$

$$\left(\frac{1}{1!}\right)q8{\cdot}(2^8-1)+\left(\frac{1}{3!}\right)q6{\cdot}(2^6-1)+\left(\frac{1}{5!}\right)q4{\cdot}(2^4-1)+\left(\frac{1}{7!}\right)q2{\cdot}(2^2-1))-\left(\frac{1}{8!}\right){\cdot}q1{\cdot}(2^1-1)=0$$

$$\left(\frac{2}{1!}\right)q8{\cdot}(2^8-1)+\left(\frac{3}{3!}\right)q6{\cdot}(2^6-1)+\left(\frac{5}{5!}\right)q4{\cdot}(2^4-1)+\left(\frac{7}{7!}\right)q2{\cdot}(2^2-1))-\left(\frac{8}{8!}\right){\cdot}q1{\cdot}(2^1-1)=0$$

We reduce the fractions, and get **the mirrorequation nr. 2**:

$$\left(\frac{2}{1!}\right)q8{\cdot}(2^8-1)+\left(\frac{1}{2!}\right)q6{\cdot}(2^6-1)+\left(\frac{1}{4!}\right)q4{\cdot}(2^4-1)+\left(\frac{1}{6!}\right)q2{\cdot}(2^2-1))-\left(\frac{1}{7!}\right){\cdot}q1{\cdot}(2^1-1)=0$$

We multiplicate our new **mirrorequation nr. 2** with $2\cdot 8!$,
We get:

$$2{\cdot}\left(\frac{8!}{1!}\right)2q8{\cdot}(2^8-1)+\left(\frac{8!}{2!}\right)2q6{\cdot}(2^6-1)+\left(\frac{8!}{4!}\right)2q4{\cdot}(2^4-1)+\left(\frac{8!}{6!}\right)2q2{\cdot}(2^2-1))-\left(\frac{8!}{7!}\right){\cdot}2q1{\cdot}(2^1-1)=0$$

We divide and multiplicate each term with the same number and get:

$$2\left(\frac{8!}{8!0!}\right)8!2q8(2^8-1)+\left(\frac{8!}{6!2!}\right)6!2q6(2^6-1)+\left(\frac{8!}{4!4!}\right)4!2q4(2^4-1)+\left(\frac{8!}{6!2!}\right)2!2q2(2^2-1)$$
$$-\left(\frac{8!}{7!1!}\right)1!2q1(2^1-1)=0$$

Note the little trick in the first term: we want the fraction to be a **Pascal coefficient** and $1! = 0!$

and now we can insert the first 4 Genocchi numbers: $g2 = 2 \cdot 2!(2^2 - 1)q2$, $g4 = 2 \cdot 4!(2^4 - 1)q4$, $g6 = 2 \cdot 6!(2^6 - 1)q6$ and $g8 = 2 \cdot 8!(2^8 - 1)q8$
we get:

$$2\left(\frac{8!}{0!8!}\right)g8 + \left(\frac{8!}{2!6!}\right)g6 + \left(\frac{8!}{4!4!}\right)g4 + \left(\frac{8!}{6!2!}\right)g2 - 8$$

or

$$2 \cdot g8 = -\left(\left(\frac{8!}{2!6!}\right)g6 + \left(\frac{8!}{4!4!}\right)g4 + \left(\frac{8!}{6!2!}\right)g2 - 8\right)$$

The **Pascal coefficients** are whole numbers.
Genocchi's 4. number multiplied with 2 is minus the sum of all earlier found Genocchi-numbers, each multiplied with a whole number plus $-2 \cdot 4$.
We now have Genocchi's 4. number multiplied with 2 is a whole number.
If all terms were even numbers, then Genocchi's 4. number must be whole.
It can be demonstrated, that the fraction in **a middle term**:$\left(\frac{8!}{4!4!}\right)$ is even and the last term is even. But the other fractions are not consistent even.
If Genocchi's numbers were consistent uneven, we could pair the terms with the same **Pascal fractions**:

$$\left(\frac{8!}{2!6!}\right)g6 + \left(\frac{8!}{6!2!}\right)g2$$

$$\left(\frac{8!}{2!6!}\right)(g6 + g2)$$

The sum of 2 uneven numbers is even: $2 \cdot n + 1 + 2 \cdot p + 1 = 2 \cdot (n + p) + 2$.
You could object: so is the sum of 2 even numbers.
But I have not met an even Genocchi number.
They can't be consistent even.
But there is no reason to despair:).
We do have, that $9 \cdot g8$ is a whole number. And if $2 \cdot g8$ is a whole number, so is $4 \cdot 2 \cdot g8$.

$$9 \cdot g8 - 8 \cdot g8 = g8$$

A whole number minus a whole number must be whole.:)

Q.E.D.

And the equation for Genocchi's 5. number:

$$2\left(\frac{10!}{0!10!}\right)g10 + \left(\frac{10!}{2!8!}\right)g8 + \left(\frac{10!}{4!6!}\right)g6 + \left(\frac{10!}{6!4!}\right)g4 + \left(\frac{10!}{8!2!}\right)g2 - 10 = 0$$

$$11 \cdot g10 - 10 \cdot g10 = g10$$

And for Genocchi's n. number:

$$2 \cdot \frac{(2 \cdot n)!}{0!(2n)!} \cdot g(2 \cdot n) + \frac{(2 \cdot n)!}{2!(2n-2)!} \cdot g(2 \cdot n - 2) + \cdots + \frac{(2 \cdot n)!}{(2n-4)!4!} \cdot g4 + \frac{(2 \cdot n)!}{(2n-2)!2!} \cdot g2 + - \frac{(2 \cdot n)!}{(2n-1)!0!} = 0$$

$$(2\cdot n+1)\cdot g(2\cdot n)-(2\cdot n)\cdot g(2\cdot n)=g(2\cdot n)$$

We are through at last, the Genocchi numbers are whole numbers, but we must demonstrate, that they are all uneven.
By the way if n is even $g(2n)$ is negative and else $g(2n)$ is positive, but that's no big deal.

They are defined by the q-values. $g(2n)=(2n)!\cdot 2\cdot q(2n)\cdot(2^{2n}-1)$
In an earlier work I demonstrated that, if n is an even number $q(2n)$ is negative and else positive.

1.1.7 Genocchi's numbers are consistent uneven

It is with the **the equation of the decreasing numerators with even numerators in all fractions eccept the first and the last** possible to demonstrate, that, if $g8$ is a whole number, it is uneven.

$$\left(\frac{1}{1!}\right)q8\cdot(2^8-1)+\left(\frac{2}{3!}\right)q6\cdot(2^6-1)+\left(\frac{4}{5!}\right)q4\cdot(2^4-1)+\left(\frac{6}{7!}\right)q2\cdot(2^2-1))-\left(\frac{7}{8!}\right)\cdot q1\cdot(2^1-1)=0$$

$$\left(\frac{1}{1!}\right)q8\cdot(2^8-1)=-\left(\left(\frac{2}{3!}\right)q6\cdot(2^6-1)+\left(\frac{4}{5!}\right)q4\cdot(2^4-1)+\left(\frac{6}{7!}\right)q2\cdot(2^2-1))-\left(\frac{7}{8!}\right)\cdot q1\cdot(2^1-1)\right)$$

We express $9\cdot g8$ with all the earlier found Genocchi numbers:

$$9\cdot g8=-\left(\frac{9!}{3!\cdot 6!}\cdot 2\cdot g6+\frac{9!}{5!\cdot 4!}\cdot 4\cdot g4+\frac{9!}{7!\cdot 2!}\cdot 6\cdot g2-9\cdot 7\right)$$

or so:

$$g8=-2\cdot\left(\frac{9!}{3!\cdot 6!}\cdot 1\cdot g6+\frac{9!}{5!\cdot 4!}\cdot 2\cdot g4+\frac{9!}{7!\cdot 2!}\cdot 3\cdot g2\right)\cdot\frac{1}{9}+7$$

The parenthesis must be divisible with 9, because we have just demonstrated g8 is a whole number.

$$g8=-2\cdot\left(\frac{8!}{3!\cdot 6!}\cdot 1\cdot g6+\frac{8!}{5!\cdot 4!}\cdot 2\cdot g4+\frac{8!}{7!\cdot 2!}\cdot 3\cdot g2\right)+7$$

g8 is an even number plus an uneven, which makes an uneven number: $-2\cdot n+2\cdot p+1=2\cdot(p-n)+1$ or $2\cdot n+2\cdot p+1=2\cdot(p+n)+1$

Q.E.D.

It's the parenthesis, which is an whole number. We must be careful here. The fractions are no longer Pascal's coefficients. The sum of the number before ! in the denominator is 9 but in the numerator is only 8.

Example $\frac{8!}{3!\cdot 6!} = 9\frac{1}{3}$.
The 1. term gets whole only because $g6 = -3$.

$$g10 = -2 \cdot \left(\frac{10!}{3! \cdot 8!} \cdot 1 \cdot g8 + \frac{10!}{5! \cdot 6!} \cdot 2 \cdot g6 + \frac{10!}{7! \cdot 4!} \cdot 3 \cdot g4 + \frac{10!}{9! \cdot 2!} \cdot 4 \cdot g2 \right) + 9$$

$$g(2 \cdot n) = -2 \cdot \left(\frac{(2 \cdot n)!}{3!(2n-2)!} 1 \cdot g(2n-2) + \cdots + \frac{(2 \cdot n)!}{(2n-3)!4!} (n-2) g4 + \frac{(2 \cdot n)!}{(2n-1)!2!} (n-1) g2 \right) + (2n-1)$$

1.1.8 Pascal's coefficients

$$\left(\frac{n!}{p! \cdot (n-p)!} \right)$$

In the fractions, we call **Pascal's coefficients**, the sum of the numbers before the ! sign: $p + (n-p)$ in the denominator always equals the number before the ! sign in the numerator:.

Demonstrate these fractions are always whole numbers.

Given

$$\frac{(4 \cdot n)!}{(2 \cdot n)! \cdot (2 \cdot n)!}$$

is a whole number

Demonstrate

$$\frac{(4 \cdot n + 1)!}{(2 \cdot n + 1)! \cdot (2 \cdot n)!}$$

is then a whole number
We can divide the product $4 \cdot n!$ in 2 parts : the $2 \cdot n$ smallest factors and the $2 \cdot n$ greatest factors.

$$\frac{(2 \cdot n)!}{(2 \cdot n)!} \cdot (2 \cdot n + 1) \cdot (2 \cdot n + 2) \cdots (4 \cdot n - 1) \cdot (4 \cdot n) \cdot \frac{1}{(2 \cdot n)!} = \frac{(4 \cdot n)!}{(2 \cdot n)! \cdot (2 \cdot n)!}$$

We can multiply the numerator and the denominator with $(2 \cdot n + 1)$.
That will not change anything.

$$\frac{(2 \cdot n)!}{(2 \cdot n + 1)!} \cdot (2 \cdot n + 1) \cdot (2 \cdot n + 2) \cdots (4 \cdot n - 1) \cdot (4 \cdot n) \cdot (2 \cdot n + 1) \cdot \frac{1}{(2 \cdot n)!} = \frac{(4 \cdot n)!}{(2 \cdot n)! \cdot (2 \cdot n)!}$$

or

$$\frac{(2 \cdot n + 1)!}{(2 \cdot n + 1)!} \cdot (2 \cdot n + 2) \cdot (2 \cdot n + 3) \cdots (4 \cdot n - 1) \cdot (4 \cdot n) \cdot (2 \cdot n + 1) \cdot \frac{1}{(2 \cdot n)!} = \frac{(4 \cdot n)!}{(2 \cdot n)! \cdot (2 \cdot n)!}$$

We multiply both sides of the sign of equation with $\frac{(4\cdot n+1)}{(2\cdot n+1)}$.

$$\frac{(2\cdot n+1)!}{(2\cdot n+1)!}\cdot(2\cdot n+2)\cdot(2\cdot n+3)\cdots(4\cdot n-1)\cdot(4\cdot n)\cdot(2\cdot n+1)\frac{1}{(2\cdot n)!}\cdot\frac{(4\cdot n+1)}{(2\cdot n+1)}$$
$$=\frac{(4\cdot n)!}{(2\cdot n)!\cdot(2\cdot n)!}\cdot\frac{(4\cdot n+1)}{(2\cdot n+1)}$$

or:

$$\frac{(2\cdot n+1)!}{(2\cdot n+1)!}\cdot(2\cdot n+2)\cdot(2\cdot n+3)\cdots(4\cdot n-1)\cdot(4\cdot n)\cdot(2\cdot n+1)\cdot\frac{(4\cdot n+1)}{(2\cdot n+1)}\cdot\frac{1}{(2\cdot n)!}$$
$$=\frac{(4\cdot n+1)!}{(2\cdot n)!\cdot(2\cdot n+1)!}$$

You will get a greater but still a whole number.

$$\frac{(2\cdot n+1)!}{(2\cdot n+1)!}\cdot(2\cdot n+2)\cdot(2\cdot n+3)\cdots(4\cdot n-1)\cdot(4\cdot n)\cdot(4\cdot n+1)\cdot\frac{1}{(2\cdot n)!}=\frac{(4\cdot n+1)!}{(2\cdot n)!\cdot(2\cdot n+1)!}$$

$\frac{(2\cdot n+1)!}{(2\cdot n+1)!}=1$ gives

$$(2\cdot n+2)\cdot(2\cdot n+3)\cdots(4\cdot n-1)\cdot(4\cdot n)\cdot(4\cdot n+1)\cdot\frac{1}{(2\cdot n)!}=\frac{(4\cdot n+1)!}{(2\cdot n)!\cdot(2\cdot n+1)!}$$

a whole, but greater number.
We have only replaced $(2\cdot n+1)$ with $(4\cdot n+1)$).
When going from the product $\frac{(4\cdot n)!}{(2\cdot n)!\cdot(2\cdot n)!}$ to the next $\frac{(4\cdot n+1)!}{(2\cdot n)!\cdot(2\cdot n+1)!}$, we only remove the first factor in the long product and add the next factor in the other end.
The number of factors in the long product will remain $2\cdot n$.
You could make a rule: if you divide $2\cdot n$ each other succeding numbers with the first $2\cdot n$ factors you will always get a whole number.
Q.E.D.

Demonstrate:

If the **middle term:** $\frac{(4\cdot n)!}{(2\cdot n)!\cdot(2\cdot n)!}$ is a whole number,

then the next **middle term:** $\frac{(4\cdot n+4)!}{(2\cdot n+2)!\cdot(2\cdot n+2)!}$ is not only whole but an even number.

In order to confirm this I will need 2 whole numbers with $(4\cdot n+3)!$ in the numerator

$\frac{(4\cdot n+3)!}{(2\cdot n+1)!\cdot(2\cdot n+1)!}$ and $\frac{(4\cdot n+3)!}{(2\cdot n)!\cdot(2\cdot n+2)!}$.

Above it is demonstrated that

$$\frac{(4 \cdot n)!}{(2 \cdot n)! \cdot (2 \cdot n)!} \cdot \frac{(4 \cdot n + 1)}{(2 \cdot n + 1)} = \frac{(4 \cdot n + 1)!}{(2 \cdot n)! \cdot (2 \cdot n + 1)!}$$

is a whole number.
Then this must be a whole number.

$$\frac{(4 \cdot n + 1)!}{(2 \cdot n)! \cdot (2 \cdot n + 1)!} \cdot 2 = \frac{(4 \cdot n + 1)!}{(2 \cdot n)! \cdot (2 \cdot n + 1)!} \cdot \frac{(4 \cdot n + 2)}{(2 \cdot n + 1)} = \frac{(4 \cdot n + 2)!}{(2 \cdot n + 1)! \cdot (2 \cdot n + 1)!}$$

We can multiply with $(4 \cdot n + 3)$ and it is still a whole number.

$$\frac{(4 \cdot n + 2)!}{(2 \cdot n + 1)! \cdot (2 \cdot n + 1)!} \cdot (4 \cdot n + 3) = \frac{(4 \cdot n + 3)!}{(2 \cdot n + 1)! \cdot (2 \cdot n + 1)!}$$

And we have the first whole number with$(4 \cdot n + 3)!$ in the numerator.

We borrow the whole number from above:

$$\frac{(4 \cdot n + 1)!}{(2 \cdot n + 1)! \cdot (2 \cdot n)!}$$

and we repeat this stealing the first number and adding a greater number in the other end, securing, it is still a whole but greater number

$$\frac{(4 \cdot n + 1)!}{(2 \cdot n + 1)! \cdot (2 \cdot n)!} \cdot \frac{(4 \cdot n + 2)}{(2 \cdot n + 2)} = \frac{(4 \cdot n + 2)!}{(2 \cdot n + 2)! \cdot (2 \cdot n)!}$$

We multiply this number with $(4 \cdot n + 3)$

$$\frac{(4 \cdot n + 2)!}{(2 \cdot n + 2)! \cdot (2 \cdot n)!} \cdot (4 \cdot n + 3) = \frac{(4 \cdot n + 3)!}{(2 \cdot n + 2)! \cdot (2 \cdot n)!}$$

And we have the second number with $(4 \cdot n + 3)!$ in the numerator.
We want these 2 numbers to demonstrate, that the next **middle term:** $\frac{(4 \cdot n+4)!}{(2 \cdot n+2)! \cdot (2 \cdot n+2)!}$ is a whole and even number.

We will now construct 2 even numbers.
1. number:

$$\frac{(4 \cdot n + 3)!}{(2 \cdot n + 1)! \cdot (2 \cdot n + 1)!} \cdot \frac{(4 \cdot n + 4)}{(2 \cdot n + 2)}$$

The first fraction is a whole number. The second fraction is 2. All in all a whole and even number.
2. number:

$$\frac{(4 \cdot n + 3)!}{(2 \cdot n + 2)! \cdot (2 \cdot n)!} \cdot \frac{(4 \cdot n + 2)}{(2 \cdot n + 1)}$$

The first fraction is a whole number. The secon fraction is 2. All in all a whole even number.

We will let $(2 \cdot n + 2)$ and $(2 \cdot n + 1)$ switch fraction, in order to give the first fraction in both numbers the same denominator.
2. number:

$$\frac{(4 \cdot n + 3)!}{(2 \cdot n + 1)! \cdot (2 \cdot n + 1)!} \cdot \frac{(4 \cdot n + 2)}{(2 \cdot n + 2)}$$

We are now able to subtract the 2 numbers: If you subtract 2 even numbers, you get an even number:

$$(2 \cdot n - 2 \cdot p) = 2 \cdot (n - p)$$

Where $p < n$

$$\frac{(4 \cdot n + 3)!}{(2 \cdot n + 1)! \cdot (2 \cdot n + 1)!} \cdot \left(\frac{(4 \cdot n + 4)}{(2 \cdot n + 2)} - \frac{(4 \cdot n + 2)}{(2 \cdot n + 2)}\right) = \frac{(4 \cdot n + 3)!}{(2 \cdot n + 1)! \cdot (2 \cdot n + 1)!} \cdot \left(\frac{2}{(2 \cdot n + 2)}\right)$$

$$= \frac{(4 \cdot n + 3)!}{(2 \cdot n + 1)! \cdot (2 \cdot n + 1)!} \cdot \left(\frac{2}{(2 \cdot n + 2)} \cdot \frac{(2 \cdot n + 2)}{(2 \cdot n + 2)}\right) = \frac{(4 \cdot n + 3)!}{(2 \cdot n + 1)! \cdot (2 \cdot n + 1)!} \cdot \left(\frac{1}{(2 \cdot n + 2)} \cdot \frac{(4 \cdot n + 4)}{(2 \cdot n + 2)}\right)$$

$$= \frac{(4 \cdot n + 3)!}{(2 \cdot n + 1)! \cdot (2 \cdot n + 1)!} \cdot \left(\frac{(4 \cdot n + 4)}{(2 \cdot n + 2) \cdot (2 \cdot n + 2)}\right) = \frac{(4 \cdot n + 4)!}{(2 \cdot n + 2)! \cdot (2 \cdot n + 2)!}$$

the next **the middle term:** $\frac{(4 \cdot n+4)!}{(2 \cdot n+2)! \cdot (2 \cdot n+2)!}$ is an even number.

Q.E.D.

The binomial formula and the triangle of Pascal

Why the name Pascal's coeffients?
I actually called them, what might be translated to my before-set fractions. But lately I have been reading this brillant book by Eli Maor: 'e: a story of a number', 1994, and in which he expand the $e = (1 + \frac{1}{n})^n = 1+1+1/2!+1/3!+1/4!+...$ with the help of the binomial formula.
There I met my fractions again as the coefficients of the terms.
These coefficients must be whole numbers, because they are the sum of the 2 coefficients in the line above in the triangle of Pascal. Ok:).
And the middle terms? are they even? Yes. When n in the $(a + b)^n$ is uneven, we will get a line with no middle term, but instead 2 equal terms and the sum of their coefficients will in the next line in the triangle give the middle term its even coefficient.

$$(a + b)^{2 \cdot n - 1} = \cdots + \frac{(2 \cdot n - 1)!}{(n - 1)! \cdot n!} \cdot a^n \cdot b^{n-1} + \frac{(2 \cdot n - 1)!}{n! \cdot (n - 1)!} \cdot a^{n-1} \cdot b^n + \cdots$$

$$(a + b)^{2 \cdot n} = \cdots + \cdots + \cdots + \frac{(2 \cdot n)!}{n! \cdot n!} \cdot a^n \cdot b^n + \cdots + \cdots + \cdots$$

$$\frac{(2\cdot n-1)!}{(n-1)!\cdot n!}+\frac{(2\cdot n-1)!}{n!\cdot(n-1)!}=\frac{(2\cdot n-1)!+(2\cdot n-1)!}{n!\cdot(n-1)!}=\frac{2\cdot(2\cdot n-1)!}{n!\cdot(n-1)!}$$
$$=\frac{2\cdot(2\cdot n-1)!\cdot n}{n!\cdot(n-1)!\cdot n}=\frac{(2\cdot n-1)!\cdot 2\cdot n}{n!\cdot(n-1)!\cdot n}=\frac{(2\cdot n)!}{n!\cdot n!}$$

They are the sum of the 2 coefficients in the line above in the triangle of Pascal.

$$(a+b)^{2\cdot n-1}=\cdots+\frac{(2\cdot n-1)!}{(2\cdot n-1-(k-1))!\cdot(k-1)!}\cdot a^{2\cdot n-1-(k-1)}\cdot b^{k-1}+\frac{(2\cdot n-1)!}{(2\cdot n-1-k)!\cdot k!}\cdot a^{2\cdot n-1-k}\cdot b^{k}+\cdots$$

$$(a+b)^{2\cdot n}=\cdots+\cdots+\cdots+\frac{(2\cdot n)!}{(2\cdot n-k)!\cdot k!}\cdot a^{2\cdot n-k}\cdot b^{k}+\cdots+\cdots+\cdots$$

$$\frac{(2\cdot n-1)!}{(2\cdot n-1-(k-1))!\cdot(k-1)!}+\frac{(2\cdot n-1)!}{(2\cdot n-1-k)!\cdot k!}=(2\cdot n-1)!\frac{1}{(2\cdot n-1-(k-1))!\cdot(k-1)!}+\frac{1}{(2\cdot n-1-k)!\cdot k!}$$

$$=(2\cdot n-1)!\cdot\frac{(2\cdot n-1-(k-1))!\cdot(k-1)!+(2\cdot n-1-k)!\cdot k!}{(2\cdot n-1-(k-1))!\cdot(k-1)!\cdot(2\cdot n-1-k)!\cdot k!}$$

$$=(2\cdot n-1)!\cdot(2\cdot n-1-k)!\cdot\frac{(2\cdot n-1-(k-1))\cdot(k-1)!+k!}{(2\cdot n-1-(k-1))!\cdot(k-1)!\cdot(2\cdot n-1-k)!\cdot k!}$$
$$=(2\cdot n-1)!\cdot(2\cdot n-1-k)!\cdot(k-1)!\cdot\frac{(2\cdot n-1-(k-1))+k}{(2\cdot n-1-(k-1))!\cdot(k-1)!\cdot(2\cdot n-1-k)!\cdot k!}$$
$$=(2\cdot n-1)!\cdot(2\cdot n-1-k)!\cdot(k-1)!\cdot\frac{2\cdot n}{(2\cdot n-1-(k-1))!\cdot(k-1)!\cdot(2\cdot n-1-k)!\cdot k!}$$
$$=(2\cdot n-1)!\cdot\frac{2\cdot n}{(2\cdot n-1-(k-1))!\cdot k!}=\frac{(2\cdot n)!}{(2\cdot n-k)!\cdot k!}$$

and at the top of Pascals triangle we have:

$$(a+b)=1\cdot a+1\cdot b$$

$$(a+b)^2=1\cdot a^2+2\cdot a\cdot b+1\cdot b^2$$
$$(a+b)^3=1\cdot a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+1\cdot b^3$$

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